

# Driven/Active Transport of Magnetic Particles in Microfluidic Environments

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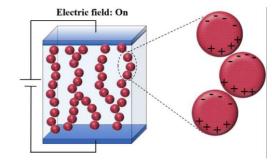
Max Planck Institute for Dynamics and Self-Organization, Germany

### **1. Driven Annealing of Magnetic Colloid**

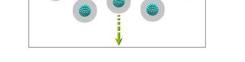
### Solid-Liquid Colloid



#### **External-Field** cued colloid



Adapted from Nanomater. 5, 2249 (2015)



Laser on

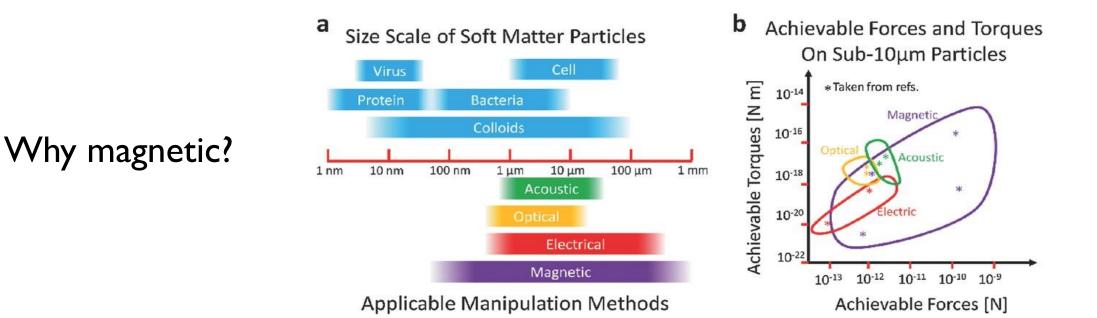
Adapted from Sci. Adv. 3, e1700458 (2017)

Adapted from PNAS 115 , 10618 (2018)

100 μm

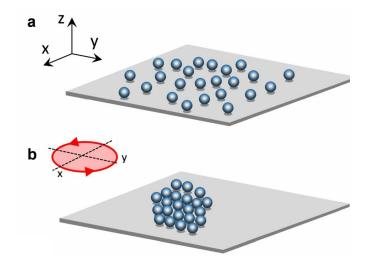
Adapted from Nat. Comm. 7, 10694 (2016)

### **External-Field Cued Soft Matter Particles**



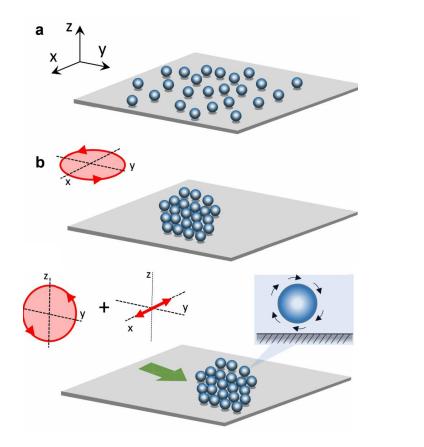
Erb et al. Adv. Funct. Mater, 26, 3859 (2016)

### Phenomenon

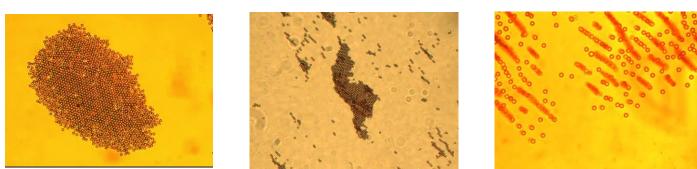


**Magnetic field**  $\boldsymbol{B} = B_0 [\cos{(\omega t)} \hat{\boldsymbol{x}} - \sin{(\omega t)} \hat{\boldsymbol{y}}]$ 

### Phenomenon



Increasing  $B_z$ 

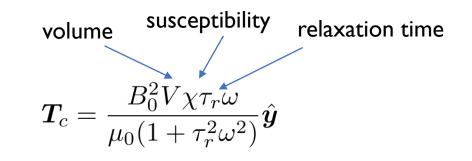


#### Why this and how to control

**Magnetic field**  $\boldsymbol{B} = B_0[\sin(\omega_x t)\hat{\boldsymbol{x}} + \cos(\omega t)\hat{\boldsymbol{y}} - (B_z/B_0)\sin(\omega t)\hat{\boldsymbol{z}}]$ 

### **Individual Paramagnetic Particle**

#### Magnetic torque



Magnetic field  $\boldsymbol{B} = B_0 [\cos{(\omega t)} \hat{\boldsymbol{x}} - \sin{(\omega t)} \hat{\boldsymbol{z}}]$ 

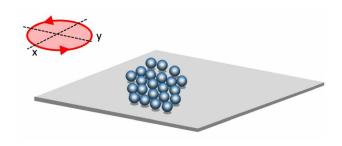
In presence of a wall in xy plane

$$\mathbf{v}_0 = \frac{T_c a^2}{32\pi\eta h^4} \hat{\boldsymbol{x}}$$

Cebers and Kalis, Euro. Phys. J. E 34, 1292 (2011)

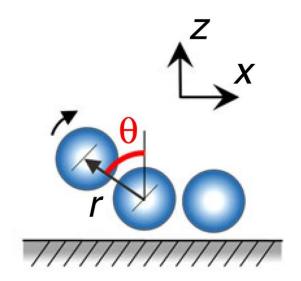
Martinez-Pedrero, Ortiz-Ambriz, Pagonabarraga, and Tierno, Phys. Rev. Lett. 115, 138301 (2015)

# Magnetic Dipole-Dipole Interaction and Hydrodynamic Interaction



Magnetic dipole-dipole interaction

$$U_{\rm m} = -\sum_{i,j\neq i} \frac{\mu_0 \left[ 3(\boldsymbol{m}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{m}_j \cdot \boldsymbol{r}_{ij}) - \boldsymbol{m}_i \cdot \boldsymbol{m}_j r_{ij}^2 \right]}{4\pi r_{ij}^5}$$



Hydrodynamic interaction

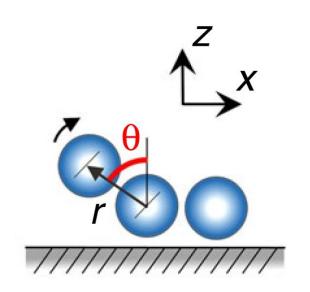
$$oldsymbol{v} = rac{oldsymbol{T} imes oldsymbol{r}}{8\pi\eta r^3} = rac{aoldsymbol{\omega}_c imes oldsymbol{\hat{r}}}{4}$$

Effective torque

$$\boldsymbol{T}_{\rm h} = 3\pi\eta a^3 \boldsymbol{\omega}_c = \frac{\pi a^3 B_0 B_z \chi \tau_r \omega}{2\mu_0 (1 + \tau_r^2 \omega^2)} \hat{\boldsymbol{y}}.$$

Massana-Cid #, Meng #, Matsunaga, Golestanian, and Tierno, Nat. Comm. 10, 2444 (2019)

### Many Particles: Effective Energy Form



Effective energy

$$U_{tot} = U_m + U_h \qquad U_h = T_h \theta$$

Dynamic equation

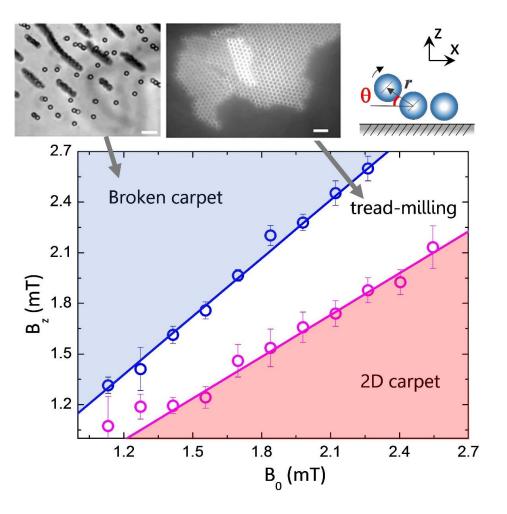
$$\dot{\theta} = \frac{1}{4\zeta a^2} \frac{3(V\chi B_0)^2}{64\mu_0 \pi a^3} \left(1 - \frac{B_z^2}{B_0^2}\right) \sin 2\theta - \frac{1}{4\zeta a^2} \frac{\pi a^3 B_0 B_z \chi \tau_r \omega}{2\mu_0 (1 + \tau_r^2 \omega^2)}$$

### Many Particles: Dynamic Regimes

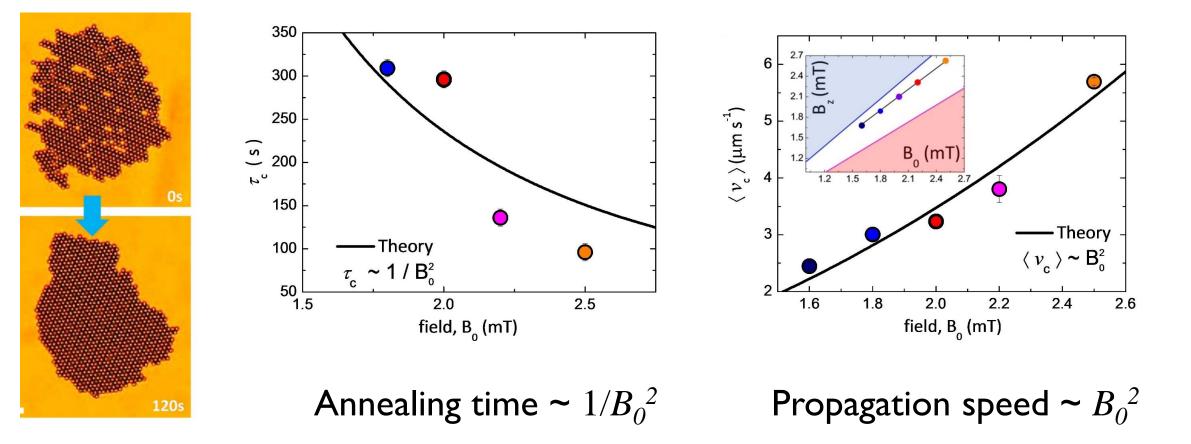
$$\frac{B_z}{B_0} \leq \frac{-c + \sqrt{c^2 + 4}}{2} \quad (\theta \sim \pi/2)$$
$$\frac{B_z}{B_0} \geq \frac{c + \sqrt{c^2 + 4}}{2} \quad (\theta \sim 0)$$

where

$$c = 6\tau_r \omega / [\chi (1 + \tau_r^2 \omega^2)]$$



### **Tread-Milling: Driven Annealing**



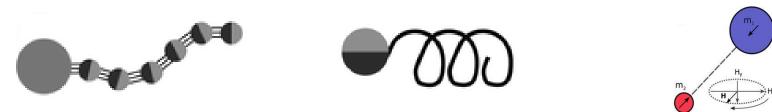
### **2. Active Clustering of Magnetic Microswimmers**

# Magnetic Microswimmer

• Natural: magnetotactic bacteria (Blakemore, Science 1975)



• Synthetic:

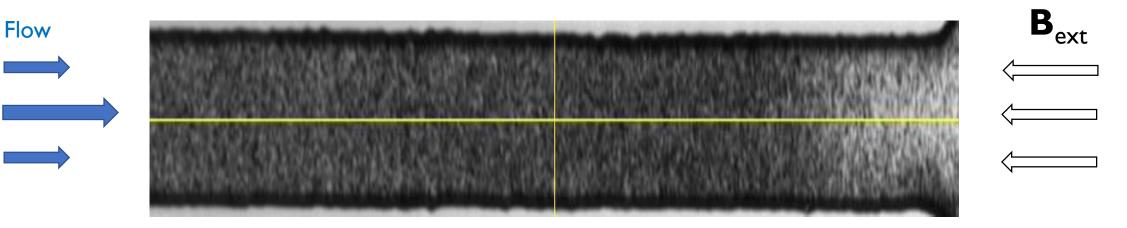


Dreyfus et al. Nature 437, 862 (2005)

Ghosh and Fischer, Nano Lett. 9, 2243 (2009)

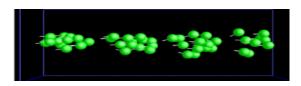
Hamilton et al. Sci. Rep. 7, 44142 (2017)

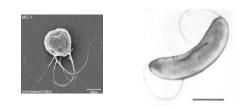
### Magnetotactic Bacteria in Microfluidic Channel



Waisbord, Lefevre, Bocquet, Ybert, Cottin-Bizonne, PRFluids 1, 053203 (2016)

#### Why clustering?



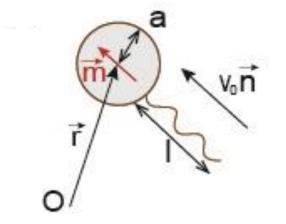


#### Pullers: attractive hydrodynamic interaction

Garcia, et.al., PRL (2013) Jibuti, et.al., PRE (2014) Lauga, et.al., EPL (2016)

#### magnetic dipole-dipole interaction

# Individual Magnetic Microswimmer



magnetotactic bacterium

Equation of motion

$$\frac{d\boldsymbol{n}}{dt} = \left[\frac{m_0\boldsymbol{n} \times (\boldsymbol{B}_{\text{ext}} + \boldsymbol{B}_{\text{int}})}{\zeta_R} + \frac{\boldsymbol{\nabla} \times \boldsymbol{V}_{\text{f}}}{2} + \boldsymbol{\xi}_R\right] \times \boldsymbol{n}$$

$$\frac{d\boldsymbol{r}}{dt} = v_0\boldsymbol{n} + \boldsymbol{V}_{\text{f}} + \frac{1}{\zeta}\boldsymbol{\nabla}(m_0\boldsymbol{n} \cdot \boldsymbol{B}_{\text{int}}) + \boldsymbol{\xi}$$

Meng, Matsunaga, Golestanian, Phys. Rev. Lett., 120, 188101 (2018)

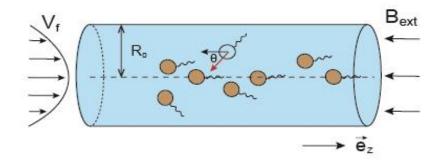
## **Orientation: Pinned**

Faster relaxation of rotation than translation

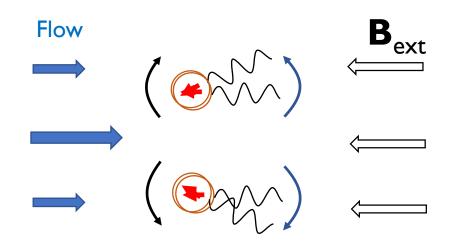
$$\frac{d\boldsymbol{n}}{dt} = \left[\frac{m_0\boldsymbol{n} \times (\boldsymbol{B}_{\text{ext}} + \boldsymbol{B}_{\text{int}})}{\zeta_R} + \frac{\boldsymbol{\nabla} \times \boldsymbol{V}_{\text{f}}}{2} + \boldsymbol{\xi}_R\right] \times \boldsymbol{n} \equiv \boldsymbol{O}$$

Swimmers are pinned in orientations, depending on the radial positions

$$\sin\theta \simeq \frac{v_{\rm f}k_{\rm B}Tr}{(D_r R_0^2 m_0 B_{\rm ext})}$$



# **Translation: Radial Focusing**



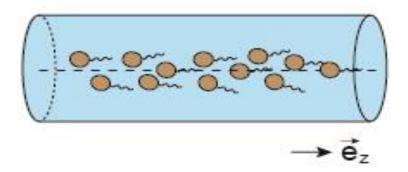
Brownian particle in a quadratic potential well

$$\dot{r} + \frac{k_B T v_0 v_{\rm f}}{m_0 B_{\rm ext} D_R R_0^2} r + \frac{k_B T v_0}{m_0 B_{\rm ext} D_R} \xi_R^{\phi} = 0$$

Number density at time t and location **r** 

$$\rho = \rho_z(z;t) \frac{R_0^2}{2R^2} \exp[-\frac{r^2}{2R^2}]$$

Focusing radius: 
$$R^2 = \frac{v_0 k_B T}{v_f m B_{ext}} R_0^2$$



### Translation: Longitudinal Clustering

Fokker-Planck equation describing the evolution of number density at time t and location  $\mathbf{r}$ 

$$\frac{\partial \langle \rho(r,z;t) \rangle_r}{\partial t} = -\nabla_z \left[ \left\langle \rho(r,z;t) v_0 \boldsymbol{n} \cdot \boldsymbol{e}_z \right\rangle_r + \left\langle \rho(r,z;t) V_{\rm f} \right\rangle_r \right] \\ \left[ -\nabla_z \left[ \frac{1}{\zeta} \left\langle \rho(r,z;t) \nabla_z (m_0 \boldsymbol{n} \cdot \boldsymbol{B}_{\rm int}) \right\rangle_r \right] + D \left\langle \nabla_z^2 \rho(r,z;t) \right\rangle_r \right] \right]$$

## **Dispersion Relation**

By assuming  $\rho_z(z;t) = \rho_z^0 + \delta \rho_z(z;t)$  and expressing the perturbation in its Fourier transformed form,  $\delta \rho_z(z;t) = \frac{1}{4\pi^2} \int \int d\omega dk_z \exp[-i\omega t + ik_z z] \delta \hat{\rho}_z(k_z,\omega)$ 

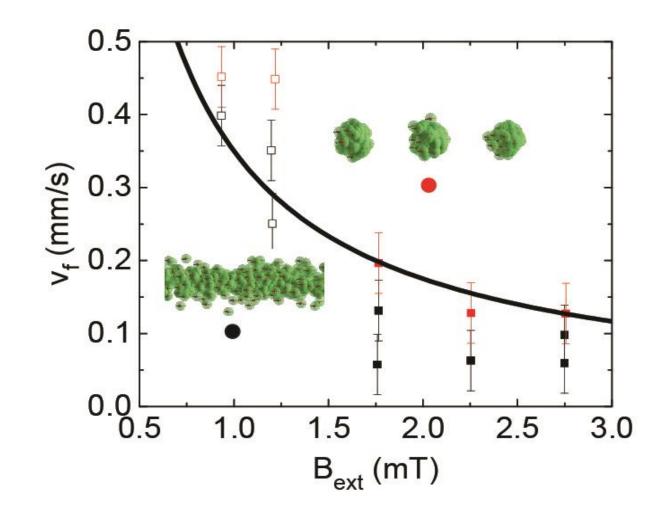
### The dispersion relation

$$-i\omega = \frac{k_z^2}{\zeta} \left[ \frac{\mu_0 \rho_0 m^2 R_0^2}{4R^2} g(k_z R) - k_B T \right] + \frac{ik_z [v_0 - v_f]}{g(q)} g(q) = \left[ 1 + q^2 \exp(q^2) \operatorname{Ei}(-q^2) \right]$$

growth rate (stability factor) propagation factor

clustering condition: 
$$\underbrace{\frac{\mu_0\rho_0m^2}{4k_BT}}_{\langle 1\rangle}\underbrace{\frac{mB_{\rm ext}}{k_BT}}_{\langle 2\rangle}\underbrace{\frac{v_{\rm f}}{v_0}}_{\langle 3\rangle} \geq 1$$

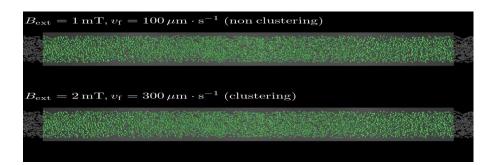
### Comparison with Experiment



clustering condition:

$$\underbrace{\frac{\mu_0 \rho_0 m^2}{4k_B T}}_{\langle 1 \rangle} \underbrace{\frac{m B_{\text{ext}}}{k_B T}}_{\langle 2 \rangle} \underbrace{\frac{v_{\text{f}}}{v_0}}_{\langle 3 \rangle} \ge 1$$

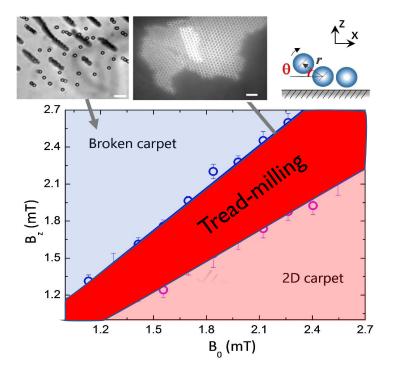
Langevin simulations:



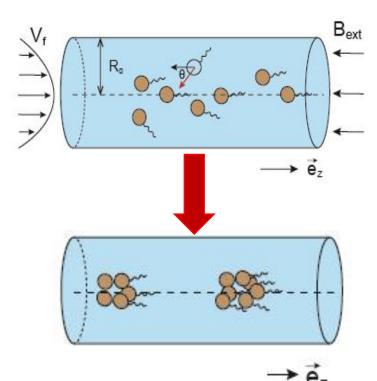
Long time responses: Cahn-Hilliard type phase transition

## Take Home Message

I. Dynamic modes of magnetic colloid



2. Clustering of magnetic microswimmers in a channel



Massana-Cid #, **Meng** #, Matsunaga, Golestanian, and Tierno, Nat. Comm. 10, 2444 (2019) **Meng**, Matsunaga, Golestanian, Phys. Rev. Lett., 120, 188101 (2018)

### Acknowledgement

#### **Collaborators:**

Helena Massana-Cid (Barcelona) Daiki Matsunaga (Osaka) Pietro Tierno (Barcelona) Ramin Golestanian (Goettingen)





### Thank you very much